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ACOUSTIC WAVE DIFFRACTION BY AN IDEAL SCREEN IN A PLANE WAVEGUIDE WITH THIN ELASTIC WALLS*

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There is considered the diffraction problem for a plane waveguide with elastic walls. The boundary conditions yielding an identical mechanical mode of the waveguide walls and containing high order derivatives are not made specific. The natural acoustic wave from the waveguide depth is the field source. Diffraction of this wave by an ideal screen with a height half the waveguide width is studied. The screen is considered either absolutely rigid (Neumann condition), or absolutely soft (Dirichlet condition). Solutions are constructed for the case when plates capable only of bending vibrations are the walls and the boundary-contact conditions needed for unique solvability of the problem /1/ are selected here so that they describe the junction of the screen and one of the plates, are constructed as examples. There is also obtained a solution for the case of impedance conditions on the waveguide walls. A similar wave diffraction problem by a diaphragm in a waveguide with ideal walls (absolutely soft or absolutely rigid) but without matrix nature, is examined in /2/.

1. Formulation of the problem. The acoustic field satisfies the homogeneous Helmholtz equation (Fig.1)

$$(\Delta + k^2)P(x, y) = 0, \quad -\infty < x < +\infty, \quad -h < y < h \tag{1.1}$$

the boundary conditions on the walls

$$L_\alpha P\,[x, (-1)^{\alpha+1}h] = 0, \quad -\infty < x < +\infty \tag{1.2}$$

$$L_\alpha = M_1\left(-\frac{\partial^2}{\partial x^2}\right)\frac{\partial}{\partial y} + (-1)^{\alpha+1} M_2\left(-\frac{\partial^2}{\partial x^2}\right), \quad \alpha = 1, 2$$

and the Dirichlet or Neumann condition on the screen

$$P(0, y) = 0, \quad -h < y < 0 \tag{1.3}$$

$$\frac{\partial P(0, y)}{\partial x} = 0, \quad -h < y < 0 \tag{1.4}$$

within the waveguide.

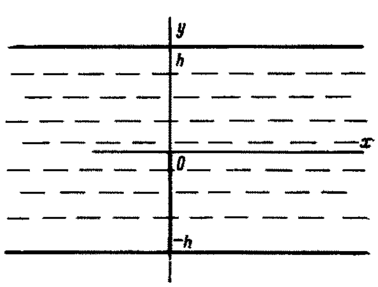

Fig.1

Here k is the wave number in the medium, M_1, M_2 are polynomials of argument $-\partial^2/\partial x^2$ whose coefficients depend on the mechanical parameters of the problem. We shall consider the order $2n_1$ of the polynomial M_1 to be higher than the order $2n_2$ of the polynomial M_2.

Let us present two examples of the realization of the operators L_α.

In the impedance case

$$L_\alpha = \frac{\partial}{\partial y} + (-1)^{\alpha+1}\eta \tag{1.5}$$

and the orders of the polynomials M_1, M_2 are zero.

If plates capable of just bending vibrations are the waveguide walls, then

$$L_\alpha = \left(\frac{\partial^4}{\partial x^4} - \varkappa^4\right)\frac{\partial}{\partial y} + (-1)^{\alpha+1}\nu, \quad \nu = \frac{\rho_0\omega^2}{D}, \quad \varkappa^4 = \frac{\rho\omega^2}{D} \tag{1.6}$$

where ρ_0, ρ are, respectively, the fluid density and the surface density of the plate, D is the cylindrical plate stiffness, and ω is the frequency of vibration. The time dependence of the wave processes is selected in the form $e^{-i\omega t}$ and will henceforth be omitted throughout.

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Let us examine diffraction by a screen in a waveguide, of a normal pressure mode, symmetric in y, and incident on the screen from the right half of the waveguide

$$P_0(x, y) = \exp(-i\lambda_0 x)\,\mathrm{ch}(\gamma_0, y), \quad \gamma = \gamma(\lambda) = \sqrt{\lambda^2 - k^2}, \quad \gamma_0 = \gamma(\lambda_0)$$

The wave number λ_0 of the normal mode is found from the condition that $P_0(x, y)$ satisfies the boundary conditions (1.2)

$$L_1 P_0(x, h) = 0$$

Let us note that $P_0(x, y)$ satisfies all the conditions of the problem except conditions (1.3) or (1.4) on the screen. The case when a normal mode anti-symmetric in y is the field source $P_0(x, y) = \exp(-i\lambda_0 y)\,\mathrm{sh}(\gamma_0 y)$ is examined analogously.

Partitioning the total field $P(x, y)$ and $P_0(x, y)$ into even or odd parts in the coordinate x

$$P(x, y) = P_s(x, y) + P_a(x, y), \quad P_s(x, y) = {}^1/_2\,[P(x, y) + P(-x, y)], \quad P_a(x, y) = {}^1/_2\,[P(x, y) - P(-x, y)]$$

we arrive at two independent problems in a semi-infinite waveguide.

The conditions

$$\frac{\partial P_s(0, y)}{\partial x} = 0, \quad -h < y < h \tag{1.7}$$

$$P_a(0, y) = 0, \quad 0 < y < h, \quad \frac{\partial P_a(0, y)}{\partial x} = 0, \quad -h < y < 0 \tag{1.8}$$

are satisfied on the endface of this semi-infinite waveguide for $x = 0$, $|y| < h$ in the case of an absolutely rigid screen in the waveguide.

We have analogous conditions for the case of an absolutely soft screen

$$P_a(0, y) = 0, \quad -h < y < h, \quad \frac{\partial P_s(0, y)}{\partial x} = 0, \quad 0 < y < h, \quad P_s(0, y) = 0, \quad -h < y < 0$$

If the order $2n_1 + 1$ of the boundary operator L_α is greater than one, then for the solution to be unique it is necessary to mention the boundary-contact conditions yielding the mechanical mode at the angular points of the semi-infinite waveguide. For the upper wall this condition is no mechanical defects in the waveguide wall, while for the lower it is the condition for fastening the diaphragm to the wall. The scattered field is constructed in conformity with the principle of limiting absorption, and should satisfy the Meixner condition "on an edge" /3/. In conformity with the method of solving boundary-contact problems taken in /4, 5/, we will seek the solution for the even and odd field components $P_s(x, y), P_a(x, y)$ in x in the form of the sum of the particular solution $R(x, y)$ of the inhomogeneous problem and the general solution $Q(x, y)$ of the homogeneous problem. The function $R(x, y)$ is found uniquely from the requirement that at the angular points it have continuous derivatives of all the orders taking part in the boundary-contact conditions.

The function $Q(x, y)$ contains discontinuities in the total field derivatives at the points $(0, \pm h)$ and describes the field diverging from these points /5/.

2. Solution of the odd problem. Let us represent the particular solution $R_{0a}(x, y)$ in the form

$$R_{0a}(x, y) = {}^1/_2\,[P_0(x, y) + (-1)^\alpha P_0(-x, y)] + R_{\alpha 0}(x, y) \tag{2.1}$$

Here and henceforth $\alpha = 1$, throughout if $0 < y < h$, and $\alpha = 2$, if $-h < y < 0$. We expand the unknown functions $R_{0a}(x, y)$ in plane waves

$$R_{\alpha a}(x, y) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} e^{i\lambda x}\,[r_{\alpha s}(\lambda)\,\mathrm{ch}(\gamma y) + r_{\alpha a}(\lambda)\,\mathrm{sh}(\gamma y)]\,d\lambda \tag{2.2}$$

The boundary conditions will be satisfied on the endface (1.8) if oddness of the functions $r_{1s}(\lambda), r_{1a}(\lambda)$ and evenness of $r_{2s}(\lambda), r_{2a}(\lambda)$ are required. Introducing the vector function $r(\lambda) = \| r_{1s}(\lambda), r_{1a}(\lambda), r_{2s}(\lambda), r_{2a}(\lambda) \|^*$ (* is the sign of transposition), we write this condition in the matrix form

$$r(\lambda) = E r(-\lambda) \tag{2.3}$$

where E is a diagonal matrix with the elements $a_{11} = a_{22} = -1,\ a_{33} = a_{44} = 1.$

The boundary conditions (1.2) result in the integral equations $(x > 0)$

$$L_\alpha R_{\alpha\alpha}[x, (-1)^{\alpha+1} h] = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} e^{i\lambda x} [(-1)^{\alpha+1} r_{\alpha s}(\lambda)\, l_s(\lambda) + r_{\alpha a}(\lambda)\, l_a(\lambda)]\, d\lambda = 0 \tag{2.4}$$

$$l_s(\lambda) = \gamma M_1(\lambda^2)\, \mathrm{sh}(\gamma h) + M_2(\lambda^2)\, \mathrm{ch}(\gamma h), \quad l_a(\lambda) = \gamma M_1(\lambda^2)\, \mathrm{ch}(\gamma h) + M_2(\lambda^2)\, \mathrm{sh}(\gamma h)$$

Because of the continuity of the field $R_{0a}(x, y)$ and its normal derivative $\frac{\partial R_{0a}(x, y)}{\partial y}$ along the waveguide middle plane $y = 0$ has the integral equations $(x > 0)$

$$\frac{1}{2\pi i} \int_{-\infty}^{+\infty} e^{i\lambda x} [r_{1s}(\lambda) - r_{2s}(\lambda)]\, d\lambda = P_0(-x, 0) \tag{2.5}$$

$$\frac{1}{2\pi i} \int_{-\infty}^{+\infty} e^{i\lambda x} \gamma\, [r_{1a}(\lambda) - r_{2a}(\lambda)]\, d\lambda = 0 \tag{2.6}$$

Equations (2.4)—(2.6) will be satisfied identically if there is required

$$G(\lambda) r(\lambda) = \Phi^+(\lambda) + f(\lambda) \tag{2.7}$$

$$G(\lambda) = \begin{Vmatrix} l_s(\lambda) & l_a(\lambda) & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & \gamma & 0 & \gamma \\ 0 & 0 & -l_s(\lambda) & l_a(\lambda) \end{Vmatrix}$$

$$f(\lambda) = \frac{1}{\lambda - \lambda_0} \| 0, 1, 0, 0 \|^*$$

$$\Phi^+(\lambda) = \| \varphi_1^+(\lambda),\ \varphi_2^+(\lambda),\ \varphi_3^+(\lambda),\ \varphi_4^+(\lambda) \|^*$$

Here the components of the vector function Φ^+ are analytic functions in the upper half-plane.

Let us eliminate the unknown vector function $r(\lambda)$ by using (2.7), (2.3) and the relationship

$$G(\lambda) r(-\lambda) = \Phi^+(-\lambda) + f^+(-\lambda)$$

obtained from (2.7) by a formal replacement of λ by $-\lambda$. The components $\Phi^+(-\lambda)$ are functions analytic in the lower half-plane of the variable $\lambda\ (\mathrm{Im}\, \lambda < 0)$.

We therefore arrive at a Riemann inhomogeneous matrix problem

$$\Phi^+(\lambda) = B(\lambda)\Phi^+(-\lambda) + B(\lambda) f(-\lambda) - f(\lambda) \tag{2.8}$$

$$B(\lambda) = G(\lambda) E G^{-1}(\lambda)$$

$$B(\lambda) = \begin{Vmatrix} -1 & 0 & 0 & 0 \\ -1/l_s(\lambda) & 0 & g(\lambda) & 1/l_s(\lambda) \\ -\gamma/l_a(\lambda) & 1/g(\lambda) & 0 & -\gamma/l_a(\lambda) \\ 0 & 0 & 0 & 1 \end{Vmatrix}, \quad g(\lambda) = \frac{l_a(\lambda)}{\gamma l_s(\lambda)}$$

to find a piecewise-analytic vector function $\Phi(\lambda)$ by means of the linear relation connecting the limit values $(\mathrm{Im}\, \lambda \to +0)$ of this function $\Phi^+(\pm\lambda)$ on the real axis of the variable λ. To solve the problem, we represent the matrix $B(\lambda)$ as the product of matrices $B_1^+(\lambda)$ and $B_2^+(-\lambda)$, whose elements will be analytic functions, respectively, in the upper and lower half-planes of λ

$$B(\lambda) = B_1^+(\lambda) B_2^+(\lambda) \tag{2.9}$$

$$B_1^+(\lambda) = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ -g^+(\lambda)\Psi_1^+(\lambda) & g^+(\lambda) & 0 & g^+(\lambda)\Psi_1^+(\lambda) \\ -\Psi_2^+(\lambda)/g^+(\lambda) & 0 & 1/g^+(\lambda) & \Psi_2^+(\lambda)/g^+(\lambda) \\ 0 & 0 & 0 & -1 \end{Vmatrix}$$

$$B_2^+(-\lambda) = \begin{Vmatrix} -1 & 0 & 0 & 0 \\ \Psi_1^+(-\lambda) & 0 & g^+(-\lambda) & -\Psi_1^+(-\lambda) \\ \Psi_2^+(-\lambda) & 1/g^+(-\lambda) & 0 & \Psi_2^+(-\lambda) \\ 0 & 0 & 0 & -1 \end{Vmatrix}$$

The matrix elements are defined on the basis of the following relationships:

$$g(\lambda) = g^+(\lambda) g^-(\lambda), \quad g^+(\lambda) = g^-(-\lambda), \quad g^+(\pm\lambda) = O(\lambda^{-1/2}), \quad |\lambda| \to +\infty$$

$$\Psi_1^+(\lambda) + \Psi_1^+(-\lambda) = -\frac{1}{g^+(\lambda)\, l_s(\lambda)}, \quad \Psi_2^+(\lambda) + \Psi_2^+(-\lambda) = -\frac{1}{g^+(-\lambda)\, l_s(\lambda)}$$

The even function $g(\lambda)$ is meromorphic, has a set of pairwise opposite simple zeroes and poles, and should be factorized, for instance, according to /3/: $\psi_{1,2}^+(\lambda)$ are the limit values $(\mathrm{Im}\,\lambda \to +0)$ of the piecewise-analytic functions $\psi_{1,2}(\lambda)$

$$\Psi_1(\lambda) = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{d\tau}{g^+(\tau)\, l_s(\tau)(\tau-\lambda)}, \quad \Psi_2(\lambda) = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{d\tau}{g^+(-\tau)\, l_s(\tau)(\tau-\lambda)}$$

Taking account of (2.9), we rewrite (2.8) in the form

$$[B_1^+(\lambda)]^{-1}\Phi^+(\lambda) - V^+(\lambda) = B_2^+(-\lambda)\Phi^+(-\lambda) + V^+(-\lambda) \tag{2.10}$$

Here $V^+(\lambda)$ is the value of the piecewise-analytic vector function $V(\lambda)$ in the upper half-plane

$$V(\lambda) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \{B_2^+(-\tau) f(-\tau) - [B_1^+(\tau)]^{-1} f(\tau)\} \frac{d\tau}{\tau-\lambda}$$

$$V^+(\lambda) - V^+(-\lambda) = B_2^+(-\lambda) f(-\lambda) + [B_1^+(\lambda)]^{-1} f(\lambda), \quad \mathrm{Im}\,\lambda \to 0$$

Both sides of (2.10) determine the general vector function $W(\lambda) = \| w_1(\lambda), w_2(\lambda), w_3(\lambda), w_4(\lambda) \|^*$ analytic in the whole complex plane in accordance with the theorem on analytic continuation. By virtue of the required continuity of the derivatives of the particular solution $R_0(x, y)$, the functions $\varphi_1^+(\lambda)$, $\varphi_4^+(\lambda)$, and $w_1(\lambda)$, $w_4(\lambda)$ besides, are zero.

Since the asymptotic

$$\varphi_2^+(\lambda) = O(\lambda^{-1}), \quad \varphi_3^+(\lambda) = O(\lambda^{-1}), \quad |\lambda| \to +\infty$$

holds, then $w_2(\lambda)$, $w_3(\lambda)$ are also zero, i.e., the vector $W(\lambda)$ is zero. Taking this into account we finally obtain from (2.10) and (2.7)

$$\begin{aligned}
& r(\lambda) = G^{-1}(\lambda)[B^+(\lambda)V^+(\lambda) + f(\lambda)] \\
& V^+(\lambda) = \| 0, v_2^+(\lambda), v_3^+(\lambda), 0 \|^* \\
& r_{\alpha s}(\lambda) = \frac{1}{2}\left[\frac{(-1)^{\alpha+1}}{\lambda-\lambda_0} - g^+(-\lambda) v_3^+(\lambda) + (-1)^{\alpha+1} g^+(\lambda) v_2^+(\lambda)\right] \\
& r_{\alpha a}(\lambda) = \frac{1}{2}\left[-\frac{1}{\gamma g(\lambda)(\lambda-\lambda_0)} - \frac{v_3^+(\lambda)}{\gamma g^+(-\lambda)} + (-1)^{\alpha+1}\frac{v_2^+(\lambda)}{\gamma g^+(\lambda)}\right] \\
& v_2^+(\lambda) = -\frac{1}{2\pi i}\lim_{\mathrm{Im}\,\lambda\to+0}\int_{-\infty}^{+\infty}\frac{d\tau}{g^+(\tau)(\tau-\lambda_0)(\tau-\lambda)} \\
& v_3^+(\lambda) = -\frac{1}{2\pi i}\lim_{\mathrm{Im}\,\lambda\to+0}\int_{-\infty}^{+\infty}\frac{d\tau}{g^+(-\tau)(\tau-\lambda_0)(\tau-\lambda)}
\end{aligned} \tag{2.11}$$

The general solution of the homogeneous problem $Q_a(x, y)$ which describes the field diverging from the points $(0, \pm h)$ will be sought in the form

$$Q_a(x, y) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} e^{i\lambda x}[q_{\alpha s}(\lambda)\,\mathrm{ch}(\gamma y) + q_{\alpha a}(\lambda)\,\mathrm{sh}(\gamma y)]\, d\lambda \tag{2.12}$$

where we assume that the functions $q_{\alpha s}(\lambda), q_{\alpha a}(\lambda)$ have the same evenness as $r_{\alpha s}(\lambda), r_{\alpha a}(\lambda)$ (2.3) and are sought by analogous means with the sole difference that (2.7) is homogeneous.

For continuity of the field in the neighborhood $(0, \pm h)$ it is sufficient to require the following estimates as $|\lambda| \to +\infty$:

$$q_{\alpha s}(\lambda) = O(\lambda^{-1-\varepsilon_1}), \quad q_{\alpha a}(\lambda) = O(\lambda^{-1-\varepsilon_2}), \quad 0 < \varepsilon_{1,2} < 1/2 \tag{2.13}$$

Taking account of (2.12), the vector function $W(\lambda)$, defined here by an equation of the type (2.10), has the form

$$W(\lambda) = \| S_n(\lambda), 0, 0, S_{n-1}(\lambda) \| ^*$$

where $S_n(\lambda)$, $S_{n-1}(\lambda)$ are polynomials of degrees n, $n-1$, where $S_n(\lambda)$ is a polynomial in odd powers of λ, as is seen from (2.7), and $S_{n-1}(\lambda)$ in even powers, and $n = \max[0, 2n_1 - 1]$. The coefficients of these polynomials are later determined from the boundary-contact conditions. The final expressions for $q_{\alpha s}(\lambda)$, $q_{\alpha a}(\lambda)$ are these:

$$q_{\alpha s}(\lambda) = \frac{1}{2}\left[\frac{S_n(\lambda) - S_{n-1}(\lambda)}{l_s(\lambda)} + (-1)^{\alpha+1} g^+(\lambda) w_2^+(\lambda) - g^+(-\lambda) w_3^+(\lambda)\right]$$

$$q_{\alpha a}(\lambda) = \frac{1}{2}\left[\frac{S_n(\lambda) + S_{n-1}(\lambda)}{l_a(\lambda)} - \frac{w_2^+(\lambda)}{\gamma g^+(-\lambda)} + (-1)^{\alpha+1} \frac{w_3^+(\lambda)}{\gamma g^+(\lambda)}\right]$$

$$w_2^+(\lambda) = \frac{1}{2\pi i} \lim_{\mathrm{Im}\,\lambda \to +0} \int_{-\infty}^{+\infty} \frac{[S_n(\tau) + S_{n-1}(\tau)]}{l_s(\tau) g^+(\tau)(\tau - \lambda)} d\tau$$

$$w_3^+(\lambda) = \frac{1}{2\pi i} \lim_{\mathrm{Im}\,\lambda \to +0} \int_{-\infty}^{+\infty} \frac{[S_n(\tau) - S_{n-1}(\tau)]}{l_s(\tau) g^+(-\tau)(\tau - \lambda)} d\tau$$

3. Solution of the even problem.

The particular solution of the inhomogeneous problem satisfying the relationships (1.1), (1.2), (1.6), and smooth with all its derivatives at the points $(0, \pm h)$, has the form

$$R_{0s}(x, y) = {}^1/_2 [P_0(x, y) + P_0(-x, y)] \tag{3.1}$$

We write the general solution of the homogeneous problem in the form

$$T_s(x, y) = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} e^{i\lambda x} [t_s(\lambda) \operatorname{ch}(\gamma y) + t_a(\lambda) \operatorname{sh}(\gamma y)] d\lambda$$

requiring evenness of the functions $t_s(\lambda)$, $t_a(\lambda)$, that are the amplitudes of the vibrations, respectively symmetrical and antisymmetrical in y, for the relations (1.6) to be satisfied identically.

The boundary conditions (1.2) for $T_s(x, y)$ result in the integral equations

$$L_\alpha T_s(x, y) = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} e^{i\lambda x} [(-1)^\alpha t_s(\lambda) l_s(\lambda) + t_a(\lambda) l_a(\lambda)] d\lambda = 0 \tag{3.2}$$

The system (3.2) will be satisfied if

$$t_s(\lambda) l_s(\lambda) = \varphi_1^+(\lambda), \quad t_a(\lambda) l_a(\lambda) = \varphi_2^+(\lambda)$$

where, as before, $\varphi_{1,2}^+(\lambda)$ is a function analytic in the upper half-plane of the variable λ. Taking account of the evenness of $t_s(\lambda)$, $t_a(\lambda)$, we obtain

$$\varphi_{1,2}^+(\lambda) = \varphi_{1,2}^+(-\lambda), \quad \mathrm{Im}\,\lambda = 0$$

Hence, taking account of the continuity of the field at the points $(0, \pm h)$, by the theorem on analytic continuation through a contour we have that $\varphi_{1,2}(\lambda)$ is a polynomial in even powers of λ of order $n-1$, $n = \max[0, 2n_1 - 1]$. We denote these polynomials by $u_s(\lambda)$, $u_a(\lambda)$, and we finally obtain

$$t_s(\lambda) = u_s(\lambda) / l_s(\lambda), \quad t_a(\lambda) = u_a(\lambda) / l_a(\lambda)$$

where the $2n_1$ coefficients of these polynomials are determined from the boundary-contact conditions.

4. Boundary-contact conditions.

If impedance boundary conditions are satisfied on the waveguide walls, i.e., L_α is the operator (1.5), then $n_1 = 0$, the polynomials $u_s(\lambda)$-$u_a(\lambda)$, $S_n(\lambda)$, $S_{n-1}(\lambda)$ are identically zero, and the solution of the problem is given by (3.1), (2.11), (2.2) and (2.1). Now, let L_α be the operator (1.6), then $n_1 = 2$, and the four coefficients for the field $Q_a(x, y)$, as much as for the field $T_s(x, y)$, must be determined.

The condition of no defects on the plate at $x = 0$ /6/ should be satisfied on the upper plate $y = +h$:

$$\frac{\partial P(-0,h)}{\partial y}=\frac{\partial P(+0,h)}{\partial y} \quad (4.1)$$

$$\frac{\partial^2 P(-0,h)}{\partial y\,\partial x}=\frac{\partial^2 P(+0,h)}{\partial y\,\partial x} \quad (4.2)$$

$$\frac{\partial^3 P(-0,h)}{\partial y\,\partial x^2}=\frac{\partial^3 P(+0,h)}{\partial y\,\partial x^2} \quad (4.3)$$

$$\frac{\partial^4 P(-0,h)}{\partial y\,\partial x^3}=\frac{\partial^4 P(+0,h)}{\partial y\,\partial x^3} \quad (4.4)$$

We have on the lower plate from the equation of diaphragm motion /7/

$$\frac{\partial P(\pm 0,-h)}{\partial y}=0, \quad \frac{\partial^2 P(\pm 0,-h)}{\partial y\,\partial x}=0 \quad (4.5)$$

Let us require compliance with conditions (4.1)—(4.5) for the x-symmetric and x-antisymmetric problems separately.

Conditions (4.2) and (4.4) are satisfied identically for the field $R_{0s}(x,y)+Q_s(x,y)$ because of the evenness of $t_s(\lambda)$, $t_a(\lambda)$, and conditions (4.1) and (4.3) have the form

$$\int_{-\infty}^{+\infty} e^{+i0\lambda}\,\gamma\lambda^n\,[t_s(\lambda)\,\mathrm{sh}(\gamma h)+t_a(\lambda)\,\mathrm{ch}(\gamma h)]\,d\lambda=\int_{-\infty}^{+\infty} e^{-i0\lambda}\,\gamma\lambda^n\,[t_s(\lambda)\,\mathrm{sh}(\gamma h)+t_a(\lambda)\,\mathrm{ch}(\gamma h)]\,d\lambda=0, \quad n=0,2 \quad (4.6)$$

Here we used the notation /6/

$$\lim_{x\to\pm 0}\int_{-\infty}^{+\infty} f(\lambda)\,e^{i\lambda x}\,d\lambda=\int_{-\infty}^{+\infty} f(\lambda)\,e^{\pm i0\lambda}\,d\lambda$$

Conditions (4.5) reduce to the equations

$$\int_{-\infty}^{+\infty} e^{+i0\lambda}\,\gamma\,[-t_s(\lambda)\,\mathrm{sh}(\gamma h)+t_a(\lambda)\,\mathrm{ch}(\gamma h)]\,d\lambda=2\pi i\gamma_0\,\mathrm{sh}(\gamma_0 h) \quad (4.7)$$

$$\int_{-\infty}^{+\infty} e^{+i0\lambda}\,\gamma\lambda^2\,[-t_s(\lambda)\,\mathrm{sh}(\gamma h)+t_a(\lambda)\,\mathrm{ch}(\gamma h)]\,d\lambda=\int_{-\infty}^{+\infty} e^{-i0\lambda}\,\gamma\lambda^2\,[-t_s(\lambda)\,\mathrm{sh}(\gamma h)+t_a(\lambda)\,\mathrm{ch}(\gamma h)]\,d\lambda=0$$

Writing the explicit form of the polynomials $u_s(\lambda)$, $u_a(\lambda)$

$$u_s(\lambda)=a+b\lambda^2, \quad u_a(\lambda)=c+d\lambda^2$$

and substituting them into (4.6) and (4.7), we obtain the system

$$aI_{0s}+bI_{2s}+cI_{0a}+dI_{2a}=0, \quad aI_{2s}+bI_{4s}+cI_{2a}+dI_{4a}=0 \quad (4.8)$$

$$-aI_{0s}-bI_{2s}+cI_{0a}+dI_{4a}=2\pi i\gamma_0\,\mathrm{sh}(\gamma_0 h); \quad -aI_{2s}-bI_{4s}+cI_{2a}+dI_{4a}=0$$

to determine the boundary-contact constants a, b, c, d. The coefficients of the system (denoted by I_{ns}, I_{na}) have the form

$$I_{ns}=\frac{1}{2\pi i}\int_{-\infty}^{+\infty} e^{+i0\lambda}\,\gamma\lambda^n\,\mathrm{sh}(\gamma h)\frac{d\lambda}{l_s(\lambda)}, \quad I_{na}=\frac{1}{2\pi i}\int_{-\infty}^{+\infty} e^{+i0\lambda}\,\gamma\lambda^n\,\mathrm{ch}(\gamma h)\frac{d\lambda}{l_a(\lambda)}$$

For $n=0.2$ the integrals I_{ns}, I_{na} converge in the ordinary sense.

Closing the contour $(-\infty,+\infty)$ in the upper half-plane, we obtain

$$I_{ns}=\sum_{N=1}^{+\infty}\left.\frac{\gamma\lambda^n\,\mathrm{sh}(\gamma h)}{l_s'(\lambda)}\right|_{\lambda=\xi_N}, \quad I_{na}=\sum_{N=1}^{\infty}\left.\frac{\gamma\lambda^n\,\mathrm{ch}(\gamma h)}{l_a'(\lambda)}\right|_{\lambda=\xi_N}$$

For a fixed frequency ω only a finite number of modes being propagated exist in the waveguide. The damping modes have the asymptotic $\xi_n=O(N)$ and the series converge.

The coefficient I_{4s} can be represented in the form

$$\frac{1}{2\pi i}\int_{-\infty}^{+\infty} e^{+i0\lambda}\,\gamma\lambda^4\,\mathrm{sh}(\gamma h)\frac{d\lambda}{l_s(\lambda)}=\frac{1}{2\pi i}\int_{-\infty}^{+\infty} e^{+i0\lambda}\,d\lambda+\frac{1}{2\pi i}\int_{-\infty}^{+\infty} e^{+i0\lambda}\,\frac{[\gamma\varkappa^4\,\mathrm{sh}(\gamma h)-\nu\,\mathrm{ch}(\gamma h)]}{l_s(\lambda)}\,d\lambda$$

Upon closure in the upper half-plane, we obtain

$$I_{4s} = \sum_{N=1}^{\infty} \frac{\varkappa^4 \gamma \operatorname{sh}(\gamma h) - \nu \operatorname{ch}(\gamma h)}{l_s'(\lambda)} \Big|_{\lambda=\xi_N}$$

The boundary-contact conditions for the X-antisymmetric part of the field are examined analogously.

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